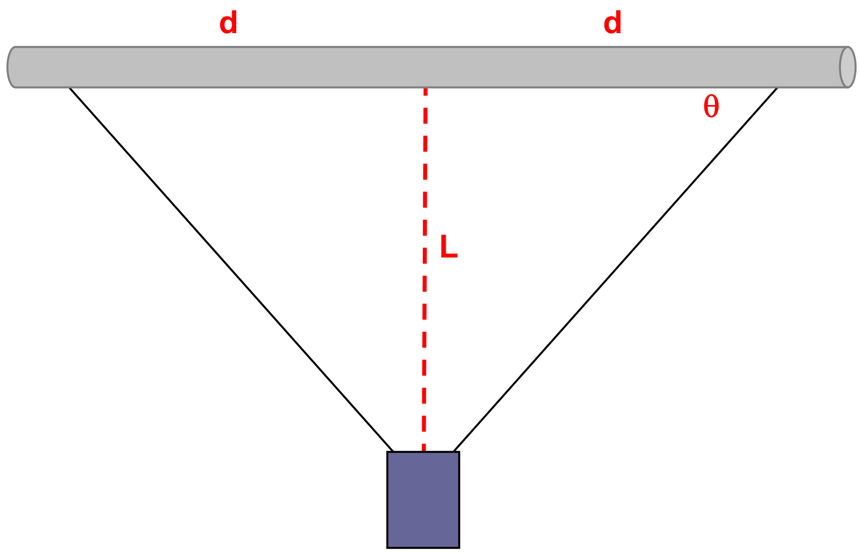
**Lab #6 Discussion**

This lab brings together numerous principles in its analysis: freebody diagrams (FBD), right triangle trigonometry, tensions and their components, conditions and equations for static equilibrium, as well as linear regression techniques. Wow! no wonder it seemed confusing and difficult!

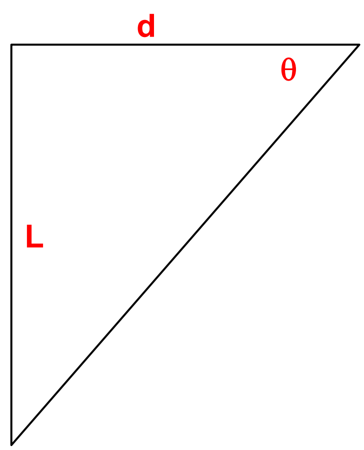
Let's begin once again examining the experimental set-up. You took numerous measurements on the tensions and heights of a mass suspended by two strings.




Once the two spring scales read the same value for each trial, you were able to determine the value of side **L** in the above right triangle by subtracting the height of the suspended mass above the table from the distance between the top of the table and the height of the horizontal supporting bar. The value of **d** was calculated by dividing the distance separating the two clamps in half.

In your analysis, you had to first determine the angle between the bar and the string, labeled θ in the diagram above. To do this, we used the trig function tangent.

The trig function tangent allowed you to calculate the value of θ.




$$\tan\theta = \frac{L}{d}$$

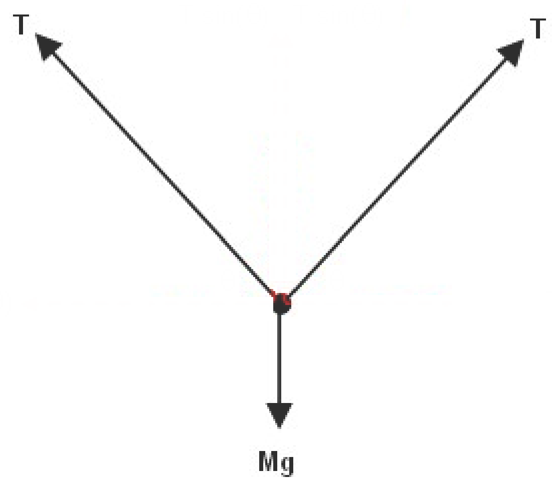
*therefore*

$$\theta = inv\tan\left(\frac{L}{d}\right)$$

*this statement means that θ is the angle whose tangent equals the ratio of L/d.*

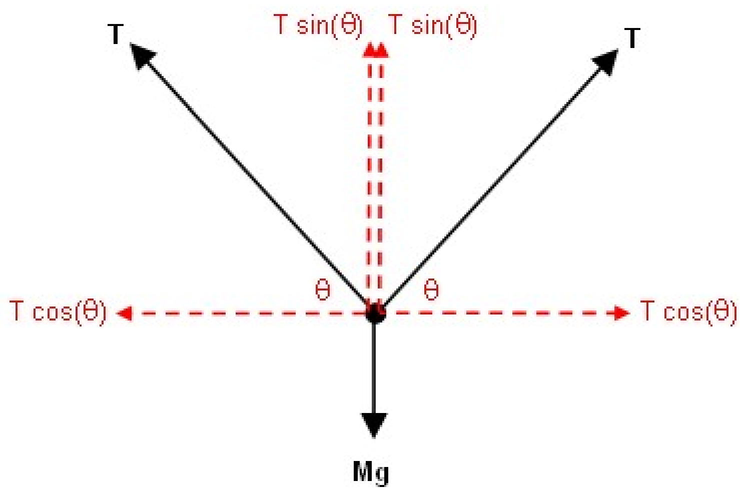
Now that we know the value of θ, we can calculate the magnitudes of the horizontal and vertical components of the tensions.

A freebody diagram of the suspended mass shows that there are three forces acting on the mass

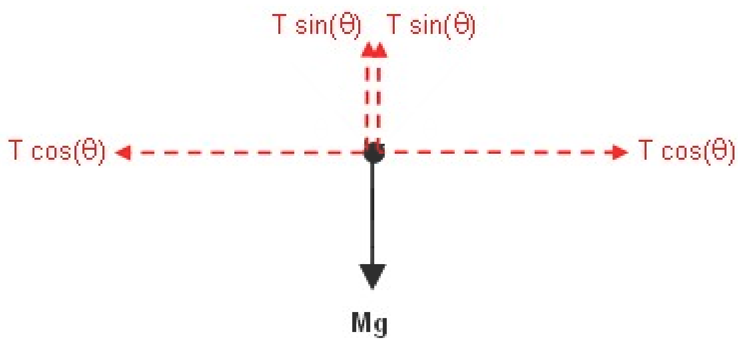



We can label each string's tension as just **T** since we know that the spring scales had the same reading in each trial.

Taking components of each string's tension produced the following diagram. Note that the left string has components bordering the negative x-axis and the positive y-axis (quadrant II) and that the right string has components bordering the positive x-axis and the positive y-axis (quadrant I).




In fact we could construct yet a third diagram only showing the components of the tensions and the weight.




It is this final force diagram that will let us set up our equations for static equilibrium.

For an object to be in equilibrium the forces acting on it must be balanced; that is the net force on the object must equal zero. Since we have forces acting in two dimensions (horizontally and vertically) we must set up two sets of equations for the net force.

$$x: \quad \text{net F}_x = 0 \qquad\qquad y: \quad \text{net F}_y = 0$$

$$T\cos\theta_{\textit{acting to the left}} = T\cos\theta_{\textit{acting to the right}} \qquad\qquad T\sin\theta_{\textit{left string}} + T\sin\theta_{\textit{right string}} = Mg$$

The equation for net $F_x$ is an identity since T and θ are the same on both strings. So this equation does not provide us with any useful information beyond the obvious knowledge that the forces are balanced.

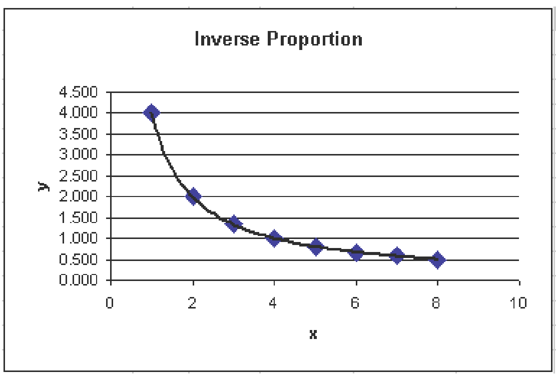
However, the equation for net $F_y$ is extremely useful and holds the solution to the lab's purpose – to experimentally determine the mass of the suspended object.

$$2T\sin(\theta) = Mg$$

$$T = \frac{Mg}{2\sin(\theta)}$$

$$T = \frac{Mg}{2}\frac{1}{\sin(\theta)}$$

This equation, when solved for **T,** tells us that the tension is <u>inversely proportional</u> to the **sine of θ**. Mathematically, a graph of **Tension vs. sin θ** would be hyperbolic.

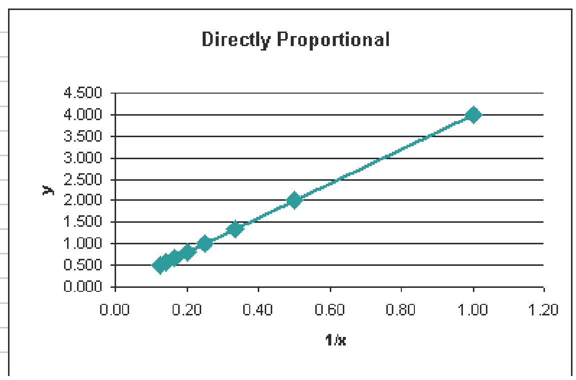



**Inverse Relations**

To convert a hyperbola to a line, change the x-axis values to 1/x as shown on the right

Do this by calculating a third data column called 1/x and then graph y vs 1/x.

The equation would read y = m(1/x) + b



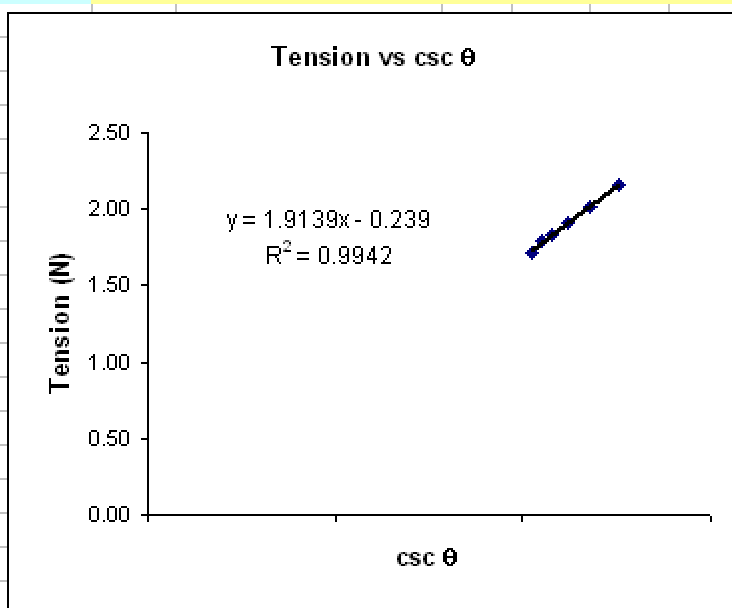

Looking at our previous equation that was solved for T, we see that $\frac{1}{\sin\theta}$ can be stated as $\csc\theta$ thus creating the equation that we need to graph

$$T = \frac{Mg}{2}\csc\theta$$

Plus the good news is, this equation will be linear!

Now we turned our attention to constructing an appropriate EXCEL data table and graph. The data used in this example is from a previous year when the students used a suspended mass of 400 grams.

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | height above the table | L | separation distance | d | tan θ | θ | csc θ | Tension |
| 2 | 0.287 | 0.64 | 0.3 | 0.15 | 4.27 | 76.81 | 1.03 | 1.71 |
| 3 | 0.299 | 0.628 | 0.4 | 0.2 | 3.14 | 72.33 | 1.05 | 1.79 |
| 4 | 0.317 | 0.61 | 0.5 | 0.25 | 2.44 | 67.71 | 1.08 | 1.83 |
| 5 | 0.338 | 0.589 | 0.6 | 0.3 | 1.96 | 63.01 | 1.12 | 1.91 |
| 6 | 0.367 | 0.56 | 0.7 | 0.35 | 1.60 | 57.99 | 1.18 | 2.01 |
| 7 | 0.396 | 0.531 | 0.8 | 0.4 | 1.33 | 53.01 | 1.25 | 2.16 |




So, what happens now?

Remember that the purpose of our lab was to experimentally determine the mass of the suspended object. Our question now becomes how can we calculate the "unknown mass" from the information on our graph?

The technique that we will use is called linear regression. We know that our data corresponds to the equation

$$T = \frac{Mg}{2}\csc\theta$$

On our EXCEL graph, **T** is being graphed on the y-axis and **csc** θ on the x-axis. If we re-examine our equation, visualizing the form y = mx + b, we see that

$$\textit{from EXCEL, if we use the variables from our x- and y-axes}$$

$$y = 1.9139x - 0.239$$

$$\textit{can be rewritten as}$$

$$T = 1.9139\csc\theta - 0.239$$

$$T = \frac{Mg}{2}\csc\theta$$

$$\textit{which tells us that the slope } 1.9139 \textit{ equals } \frac{Mg}{2}$$

$$\textit{and that our y-axis intercept should have been } 0.$$

Using this information, Mg = 3.8278 and **M = 0.390 kg**. Their experiment had a 2.5% percent error.

Let practice.

I. Complete the following table. Then open EXCEL and create a scatter plot of the last two columns, **Tension vs csc (θ)**. Finally add a trend line and the line's equation.

| | | |
|---|---|---|
| bar height above table | 0.9297 | meters |
| mass in grams | 500 | grams |

| distance between clamps (m) | height above table (m) | spring scale reading (N) | L (m) | d (m) | tan(Θ) | Θ degrees | sin(Θ) | csc(Θ) | Tension (N) |
|---|---|---|---|---|---|---|---|---|---|
| **0.3** | 0.3273 | 2.48 | | | | | | | 2.48 |
| **0.4** | 0.3431 | 2.57 | | | | | | | 2.57 |
| **0.5** | 0.3629 | 2.62 | | | | | | | 2.62 |
| **0.6** | 0.3878 | 2.79 | | | | | | | 2.79 |
| **0.7** | 0.4146 | 2.87 | | | | | | | 2.87 |
| **0.8** | 0.4528 | 3.16 | | | | | | | 3.16 |

What was the equation of the graph generated by EXCEL? ___________________________

What mathematical expression does the slope of this trend line represent? __________________________

What was the group's experimental mass in kilograms? ___________________________

What was the group's percent error? ___________________________

II. For each of the following situations, construct a FBD of both the knot and the suspended mass, M. Then draw a second FBD and resolve into components any of the diagonal tensions. Finally write the equilibrium equations for net $F_x = 0$ and for net $F_y = 0$ for both the knot and the suspended mass.